| Egypt-Japan University of Science and Technology Sample Entrance Exam (Undergraduate) |  |  |
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| Faculty of Engineering | Subject: Mathematics | (IIII |
| Academic Year: 2021/2022 | No. of Pages: 5 | E=JUST |
| Exam Duration: 1 hr | Exam Version: |  |
| Student Name: | Student ID: |  |

## Choose the correct answer:

Question (1) Which of the following graphs represents the function $x-2 y^{2}=0$ ?
a)

b)

c)

d)


Question (2) Which of the following graphs represents the function $1-x-|y|=0$ ?
a)

b)

c)

d)


Consider the two points $A(1,1,0), B(-1,1,2)$. Let $\vec{a}, \vec{b}$ be the position vectors of the two points $A, B$. Let $\vec{c}=\vec{a}+t \vec{b}$, where $t$ is a real number.

Question (3) $|\vec{c}|^{2}=$
a) $6 t^{2}+2$
b) $6 t^{2}-2$
c) $-6 t^{2}+2$
d) $-6 t^{2}-2$

Question (4) $|\vec{c}|^{2}$ is minimized at $t=$
a) $1 / 9$
b) $1 / 2$
c) 0
d) 1

Let $\alpha$ and $\beta$ be the two roots of the equation $x^{2}-p x+1=0$.
Question (5) The value of $\alpha \beta$ equals:
a) $p$
b) 1
c) $-p$
d) -1

Question (6) The value of $\int_{-2}^{2}|x| d x$ is:
a) 0
b) 1
c) 2
d) 4

Let $L$ be the tangent line to the graph $f(x)=e^{x}$ at $x=0$.
Question (7) The equation of $L$ is:
a) $y=x-1$
b) $y=2 x+1$
c) $y=x+1$
d) $y=2 x-1$

Question (8) $L$ intersects the $x$-axis at $x=$
a) -2
b) -1
c) 1
d) 2

Consider $F(x)=\int_{x}^{1}\left(3 t^{2}+1\right) d t$.
Question (9) $F(x)=$
a) $-x^{3}-x+2$
b) $x^{3}-x+2$
c) $-x^{3}+x+2$
d) $x^{3}-x-2$

Question (0) The value of $F^{\prime}(0)$ is:
a) 1
b) -1
c) 2
d) -2

Question (1) The value of $F(1)$ is:
a) 3
b) 1
c) 2
d) 0

Question (12) $\int \frac{2 x}{x^{2}+1} d x$
a) $\ln \left(x^{2}+1\right)+c$
b) $\ln \left(x^{3}+1\right)+c$
c) $\ln (2 x+1)+c$
d) $\ln \left(x^{2}+2 x\right)+c$

Question (13) $\int \frac{t}{\sqrt{t^{2}-1}} d t$
a) $\sqrt{t^{2}-1}+c$
b) $\frac{1}{\sqrt{t^{2}-1}}+c$
c) $\left(t^{2}-1\right)^{\frac{3}{2}}+c$
d) $\left(t^{2}-1\right)^{\frac{-3}{2}}+c$

Question (4) The value of $\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{k}$ is:
a) $\frac{1}{2}$
b) 2
c) 1
d) $1 \frac{1}{2}$

Question (15) A manufacturing process consists of 2 stages and each stage consists of 5 tasks. The first stage should be completed before starting the second stage. However, within any of the two stages, the tasks can be completed in any order. How many different task sequences are possible?
a) 100
b) 625
c) 10000
d) 14400

Question (16) $F_{1}=4 N, F_{2}=5 N$ are parallel forces act in the opposite direction. The distance between the second force and the resultant force equals 10 cm , then the distance between $F_{1}$ and $F_{2}$ equal.....cm.
a) 5
b) 10
c) 2.5
d) 1

Question (17) A Body has a mass of 220 kg is placed on a horizontal rough surface. The body is pulled with rob inclined to the horizontal as shown in Fig. Q. 17 at an angle of $30^{\circ}$ upwards. If the coefficient of static friction is 0.3 . What the needed tension force on the rob which makes the body is about to move? $\qquad$ kg.wt (kilogram weight).
a) 80
b) 65
c) 190.5


Fig. Q. 17

Question (18) The following system consists of 6 masses. The weight of each mass is shown in its circle. The coordinate of each mass is as shown in Fig. Q.18. Find the center of gravity of the system in $x$ and $y$ direction.
a) $\left(1 \frac{3}{8}, 0\right)$
b) $\left(\frac{3}{8}, 0\right)$
c) $\left(0, \frac{3}{8}\right)$
d) $\left(1 \frac{1}{2}, 0\right)$


Fig. Q. 18

Question (19) If the velocity of a particle is given by $=\frac{5}{2 x-4}$, calculate the acceleration " $a$ ", at displacement; $x=3$ meter
a) $\frac{2}{5}$
b) $\frac{-5}{2}$
c) $\frac{5}{4}$
d) $\frac{-25}{4}$

Question (20) A man of mass 100 kg is inside a moving lift. If the weight of the man on the floor of the lift is equal to 1500 Newton, then the lift was moving with
a) A uniform velocity not equal to zero
b) A uniform acceleration downwards.
c) A uniform acceleration upwards.
d) A uniform velocity equal to zero

## Important tables/formulas

## Differentiation Formulas:

## Integration Formulas:

1. $\frac{d}{d x}(x)=1$
2. $\int 1 d x=x+C$
3. $\frac{d}{d x}(a x)=a$
4. $\int a d x=a x+C$
5. $\frac{d}{d x}\left(x^{n}\right)=n x^{n-1}$
6. $\int x^{n} d x=\frac{x^{n+1}}{n+1}+C, n \neq-1$
7. $\frac{d}{d x}(\cos x)=-\sin x$
8. $\int \sin x d x=-\cos x+C$
9. $\frac{d}{d x}(\sin x)=\cos x$
10. $\int \cos x d x=\sin x+C$
11. $\frac{d}{d x}(\tan x)=\sec ^{2} x$
12. $\int \sec ^{2} x d x=\tan x+C$
13. $\frac{d}{d x}(\cot x)=-\csc ^{2} x$
14. $\int \csc ^{2} x d x=-\cot x+C$
15. $\frac{d}{d x}(\sec x)=\sec x \tan x$
16. $\int \sec x(\tan x) d x=\sec x+C$
17. $\frac{d}{d x}(\csc x)=-\csc x(\cot x)$
18. $\int \csc x(\cot x) d x=-\csc x+C$
19. $\frac{d}{d x}(\ln x)=\frac{1}{x}$
20. $\int \frac{1}{x} d x=\ln |x|+C$
21. $\frac{d}{d x}\left(e^{x}\right)=e^{x}$
22. $\int e^{x} d x=e^{x}+C$
23. $\frac{d}{d x}\left(a^{x}\right)=(\ln a) a^{x}$
24. $\int a^{x} d x=\frac{a^{x}}{\ln a}+C a>0, a \neq 1$
25. $\frac{d}{d x}\left(\sin ^{-1} x\right)=\frac{1}{\sqrt{1-x^{2}}}$
26. $\int \frac{1}{\sqrt{1-x^{2}}} d x=\sin ^{-1} x+C$
27. $\frac{d}{d x}\left(\tan ^{-1} x\right)=\frac{1}{1+x^{2}}$
28. $\int \frac{1}{1+x^{2}} d x=\tan ^{-1} x+C$
29. $\frac{d}{d x}\left(\sec ^{-1} x\right)=\frac{1}{|x| \sqrt{x^{2}-1}}$
30. $\int \frac{1}{|x| \sqrt{x^{2}-1}} d x=\sec ^{-1} x+C$

## Some additional integration formulas:

- $\int \frac{f^{\prime}(x)}{f(x)} d x=\ln (f(x))+C$
- $\int \frac{f^{\prime}(x)}{\sqrt{f(x)}} d x=2 \sqrt{f(x)}+C$


## Arithmetic sequence:

- General term: $\quad a_{k}=a_{1}+(k-1) d$
- Summation: $\quad S_{n}=\sum_{k=1}^{n} a_{k}=\frac{n}{2}\left(a_{1}+a_{n}\right)=\frac{n}{2}(2 a+(n-1) d)$


## Geometric sequence:

- General term:

$$
a_{k}=a_{1} r^{k-1}
$$

- Finite summation:

$$
S_{n}=\sum_{k=1}^{n} a_{k}=\frac{a_{1}\left(1-r^{n}\right)}{1-r}
$$

- Infinite summation:

$$
S_{\infty}=\sum_{k=1}^{\infty} a_{k}=\frac{a_{1}^{1-r}}{1-r}, \quad|r|<1
$$

Permutations: Number of ways of selecting $r$ out of $n$ objects taking order into consideration: $P_{r}^{n}=\frac{n!}{(n-r)!}$.

Combinations: Number of ways of selecting $r$ out if $n$ objects without taking order into consideration: $C_{r}^{n}=\frac{n!}{r!(n-r)!}$.
Mechanics:

| $\mu_{s}=\frac{F_{s}}{N}$ | $v_{\text {avg }}=\frac{\Delta s}{\Delta t}, v=\frac{d s}{d t}$ |
| :--- | :--- |
| $\tan \phi_{s}=\frac{F_{s}}{N}$ | $a_{\text {avg }}=\frac{\Delta v}{\Delta t}, a=\frac{d v}{d t}$ |
| $\mu_{k}=\frac{F_{k}}{N}$ | $v=v_{0}+a t$ |
| $\tan \phi_{k}=\frac{F_{k}}{N}$ | $v^{2}=v_{0}^{2}+2 a s$ |
| $F_{R}=\sqrt{F_{1}{ }^{2}+F_{2}{ }^{2}+2 F_{1} F_{2} \cos \theta}$ | $s=v_{0} t+\frac{1}{2} a t^{2}$ |
| $\left(F_{R}\right)_{x}=\sum F_{x},\left(F_{R}\right)_{y}=\sum F_{y}$ | momentumL $=m v$ |
| $F_{R}=\sqrt{\left(F_{R}\right)_{x}^{2}+\left(F_{R}\right)_{y}^{2}}$ | $\Delta L=m\left(v_{2}-v_{1}\right)$ |
| $\theta=\tan ^{-1} \frac{\left(F_{R}\right)_{y}}{\left(F_{R}\right)_{x}}$ | $F=m a$ |
| $\bar{x}=\frac{\int x d m}{\int d m}, \bar{y}=\frac{\int y d m}{\int d m}$, | $I=F t$ |
|  | $F t=m\left(v_{2}-v_{1}\right)$ |

