Subjective Entrance Examination	
School: **Institute of Basic and Applied Sciences**	
Duration: **1 Hour**	Date:
Course Title: **Applied and Computational Mathematics**	
No. of Questions: **10** (5 MCQ + 5 analysis)	Max. Marks: **100**
Instructor: **ACM Faculty Members**	
Allowed Equipment	Calculator, Drawing kit

Name: **ID No.**

Instructions to Students

Mobile Phones are **Not Allowed** during the Exam

- *You should **attempt** all the questions.*
- *Assume any missing data.*
- *Number of questions are 10.*
- *The exam papers are 6 pages, including this page.*
- *Select the right answers and mark it in this answer sheet page 2 and the rest of questions should be answered in the place assigned for that.*

Choose the correct answer and fill in the given table by the letter preceding the correct answer:

Part 1: Multiple Choice Questions **30 Marks**

Q1. For the expression, $f'(x) = \dfrac{4f(x+h) - f(x+2h) - 3f(x)}{2h} + LTE$,

where LTE is the local truncation error, and $LTE = O(h^p)$, then the value of p is:

a) 2 b) 1 c) 3 d) 4

Q 2. Consider the initial value problem $z' = 2x - y$, $z(0) = -1$, and using Euler's method with step size $h = 0.1$, the value of $z(0.1)$ is:

a) – 0.79 b) 0.79 c) 0.9 d) – 0.9

Q 3. If L is a self-adjoint operator on a finite-dimensional inner product space V, then every eigenvalue of L is:

a) real b) complex c) purely imaginary d) not a), b) or c)

Q 4. The Mittage-Leffler function for two parameters is given by:

a) $E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+\beta)}$ b) $E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k-\beta)}$

c) $E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^{k+1}}{\Gamma(\alpha k-\beta)}$ d) $E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\beta k+\alpha)}$

Q 5. Let $u_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, u_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, u_3 = \begin{pmatrix} -2 \\ 1 \\ -5 \end{pmatrix}, y = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$ and $y = c_1 u_1 + c_2 u_2 + c_3 u_3$, then the value of c_1 is:

a) $\frac{3}{5}$ b) $\frac{-5}{3}$ c) $\frac{-3}{5}$ d) $\frac{-7}{15}$

Answer sheet

Question No.	**1**	**2**	**3**	**4**	**5**
Answer					

Part 2: Essay and analysis questions (attempt all questions) 70 Marks

Q 6. Consider the one-dimensional parabolic PDE $u_t(x,t)=\alpha\, u_{xx}(x,t),\ 0<x<L,\ 0<t\le T$, subject to the boundary conditions $u(0,t)=\alpha_0, u(L,t)=\beta_0, 0\le t\le T$, and the initial conditions $u(x,0)=f(x),\ 0\le x\le L$.

a) Using the finite difference method, derive the classical explicit scheme.

b) Drive the necessary condition for the stability of the proposed scheme.

Q 7. Solve the system of differential equations

$$x' = 2x + 3y, \quad y' = 2x + y.$$

Q 8. Solve the initial value problem

$$u_{tt} - c^2 u_{xx} = 0, \; u(x,0) = x^3, \quad u_t(x,0) = \sin x.$$

Q 9. Solve the fractional differential equations by Laplace transform $D^{\frac{4}{3}}u(t)=0$, where D^{α} is the Riemann-Liouville fractional derivative operator (assume any missing data).

Q10. Let $t \in R$ such that t is not an integer multiple of π. For the matrix

$$A=\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix},$$

Show that there does not exist a real-valued matrix B such that BAB^{-1} is a diagonal matrix.

********************** End of Questions *************************